

# Modified Dual Compound with Modular Craft Technique

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## Abstract

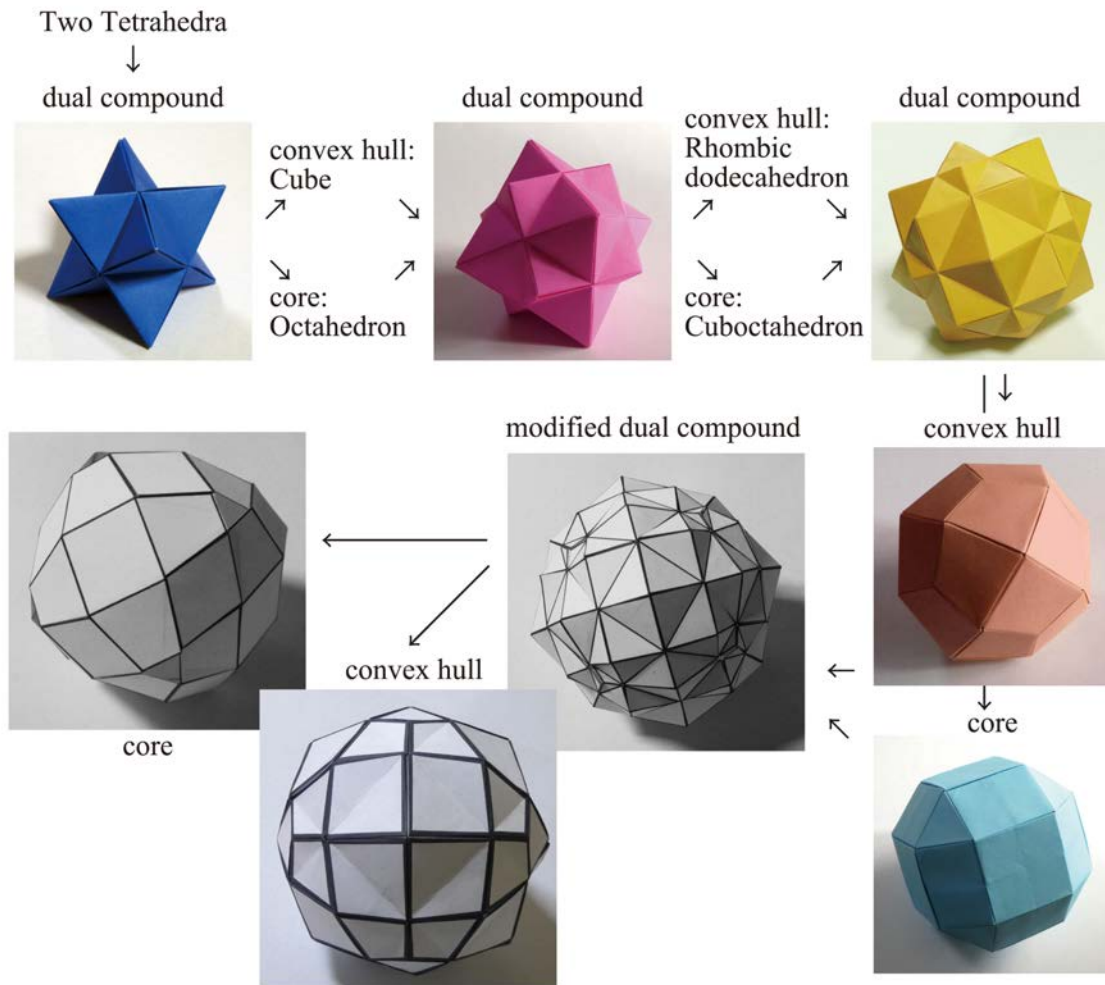
A dual compound polyhedron is made by two polyhedra dual to each other. Let us call the overlapping part of a dual compound as a core, and let us also call a polyhedron which is made by connecting all the vertices of the dual compound as a convex hull. Here we consider sets of dual compound, core and convex hull.

For example, let us consider a regular tetrahedron. A dual polyhedron of the regular tetrahedron is a regular tetrahedron. Then the dual compound is a stella octangula, the core is a regular octahedron and the convex hull is a cube. Here we define the set of these three polyhedra as the first generation of series, starting from a regular tetrahedron. Since the core and the convex hull of the first generation are dual to each other in this case, the dual compound polyhedron of the second generation can exist. The core and the convex hull of the second generation are a cuboctahedron and a rhombic dodecahedron, respectively. Then we can step toward to the third generation. The core and the convex hull of the third generation, however, are not dual to each other. Therefore the series ends with the third generation. Even if we start with another regular and semi-regular polyhedron, the series ends with the first or second generation because the core and the convex hull will not be dual any more.

Let us here define a modified dual compound, core and convex hull as follows. At first, let an arbitrary polyhedron  $c_n$  be a core of  $n$ th generation. We truncate all the vertices of  $c_n$  by some flat planes which pass through the midpoints of the edges of  $c_n$ . Let us call this truncated polyhedron as the modified core  $c_{n+1}$  of the  $(n+1)$ th generation. The faces on  $c_{n+1}$  produced by cutting off the vertices of  $c_n$  can be regarded as derived from the faces of another polyhedron  $\tilde{c}_n$ . These faces derived from  $c_n$  and  $\tilde{c}_n$  are adjacent to each other at the edges of the core  $c_{n+1}$ . (Therefore, all the vertices of the core always have even number of faces.) The edges of  $c_{n+1}$  are the dividing lines on the faces for both  $c_n$  and  $\tilde{c}_n$ . Therefore all the vertices of  $\tilde{c}_n$  are already cut off by these lines. A modified dual compound polyhedron  $C_{n+1}$  is defined as that made by all the vertices cut off from  $c_n$  and  $\tilde{c}_n$ . A  $(n+1)$ th modified convex hull  $\tilde{c}_{n+1}$  is defined as a solid whose face has a pair of diagonal lines, where one diagonal line comes from an edge of  $c_n$  and another from  $\tilde{c}_n$ . If  $\tilde{c}_n$  is the dual polyhedron of  $c_n$ , a pair of edges of  $c_n$  and  $\tilde{c}_n$  intersects each other on a flat face. Such a flat face is that of the convex hull  $\tilde{c}_{n+1}$ . On the other hand, if  $c_n$  and  $\tilde{c}_n$  are not dual to each other, a pair of edges does not intersect. Let us consider the case where a pair of edges does not intersect. From these two skew lines, we obtain the modified diagonal lines, and from those lines we

can define the modified face of  $\tilde{c}_{n+1}$ . In the case that  $c_0$  is a regular and a semi-regular polyhedron, the first generation (and sometimes the second generation as well) of the modified series is that of the unmodified dual compound series. In general, however, these modified solids  $C_{n+1}$ ,  $c_{n+1}$  and  $\tilde{c}_{n+1}$  have different shapes deformed from the unmodified ones. We can make infinitely many generations of the modified solids even when the core and the convex hull are no longer dual to each other.

Here we consider how to make models of these modified solids by using geometrical construction. All the faces of solids of  $(n+1)$ th generation can be made by scaling and rotating those of  $n$ th generation. But to make models of these solids with common modular techniques is sometimes hard because too tiny surfaces exist. Here we introduce “modular craft technique” as a new concept, which is a kind of mixture of modular technique and paper craft one. At first, we draw shapes of faces of any solid and print them out on a paper. Then we fold the paper into modules and assemble them into the solid as a modular model. Some modular craft models of unmodified and modified dual compounds, cores and convex hulls are shown in Figure.1.



**Figure 1:** An example of modified dual compound series.